DYNAMIC ANALYSIS OF THE ELASTIC DRIVESHAFTS TRANSMISSION SYSTEMS - THE EQUIVALENT STIFFNESS CALCULATION (IDEAL CASE η=0)

ANALIZA DINAMICĂ A SISTEMELOR DE TRANSMISIE CU ARBORI ELASTICI - CALCULUL RIGIDITĂŢILOR ECHIVALENTE (CAZUL IDEAL η=0)

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Abstract: *The components of the mechanical systems like machines and/or technological equipment have a certain elasticity or rigidity. Knowing the rigidity of each component is very important for the studies whose goal is to establish the dynamic efforts and the stresses in each shaft, gear wheel, steel structure, aso. The rigidity can be an important factor for the dynamic load estimation process. The components with high elasticity are the most important inducers of elastical forces and couples of force; we can enumerate: shafts, coupling gears, gears, elastical couplings, springs, long steel structures, some working devices, aso. This article presents a method and an equation involving the rigidities of the elastical shafts of the mechanical transmissions with gears in any point of the system, so that the dynamic analysis should become easier.*
Keywords: *elastical systems, equivalent rigidity, gearing, shaft stress*

Rezumat: *Componentele sistemelor mecanice precum mașinile și / sau echipamentele tehnologice au o anumită elasticitate/rigiditate. Cunoaşterea rigidității fiecărei componente este foarte importantă pentru studiile ale căror scop este de a stabili eforturile dinamice și tensiunile din fiecare ax/arbore, roată dințată, structură metalică, etc. Rigiditatea poate fi un factor important pentru procesul de estimare a încărcării dinamice. Componentele cu elasticitate ridicată sunt cei mai importanți factori ai forțelor elastice și ai momentelor elastice; dintre acestea se pot enumera: arbori, cuplaje, angrenaje, cuplaje elastice, arcuri, structuri lungi/suple din oţel, unele dispozitive de lucru, etc. Acest articol prezintă o metodă de determinare ale rigidităților echivalente ale arborilor elastici ai transmisiilor mecanice cu angrenaje în orice punct al sistemului, astfel încât analiza dinamică să poată fi aplicată pe unele modele convenţionale de calcul cunoscute.*
Cuvinte cheie: *sisteme elastice, rigiditate echivalentă, angrenaje cu roţi dinţate, solicitări torsionale în arbori elastici drepţi*

1. INTRODUCTION

The equivalent coefficient of rigidity is the mechanical feature of an equivalent elastical element (generally named spring), which replaces the real element on the basic principle of the equation of the potential energy [1] [2] [3] [4]. This means that the deformation potential energy of the equivalent element V_{eqv} is equal to the deformation potential energy of the actual element V .

2. THE EQUIVALENT RIGIDITY OF THE SHAFTS WITH ONE GEAR STEP

In order to describe the rigidity equation method, we consider a simple mechanical driving system as in figure 1, where M_M is the driving motor moment, M_{WD} is the moment of working device, **2** and **3** are the wheels of the one step gearing, k_1 and k_2 are the rigidity coefficients of the shaft **I** (driving shaft) respectively shaft **II** (driven shaft) [5] [6] [7].

It is considered that the ratio gear step is η is

$$i = \frac{\omega_{gw2}}{\omega_{gw3}} \ , \tag{1}$$

where ω_{gw2} and ω_{gw3} are the angular speeds of the wheel gear **2** respectively **3**.

The instantaneous real angular rotations of the shafts' terminations are:

-for the shaft **I** - φ_1 , φ_2

-for the shaft **II** - φ_3 , φ_4

The equivalent inertia moments of the working device and of wheel gear **3** can be calculated according to [8] [9] [10] [11].

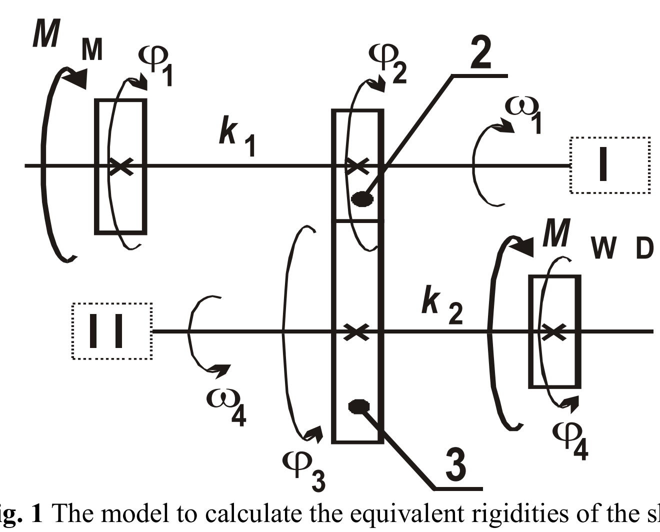

Fig. 1 The model to calculate the equivalent rigidities of the shafts

2.1.Calculus of the equivalent rigidity on the driving shaft

If the needs of equation is to be done on the shaft **I**, the fig. 2 shows the calculus model, where the significance of the notations is as follows:

► k_{2eqv} - the equivalent rigidity coefficient of the driven shaft

► φ_3^*, φ_4^* - the equivalent angular rotations of the driven shaft's terminations

► ω_1 - the average angular speed of the driving shaft (**in steady-state conditions**)

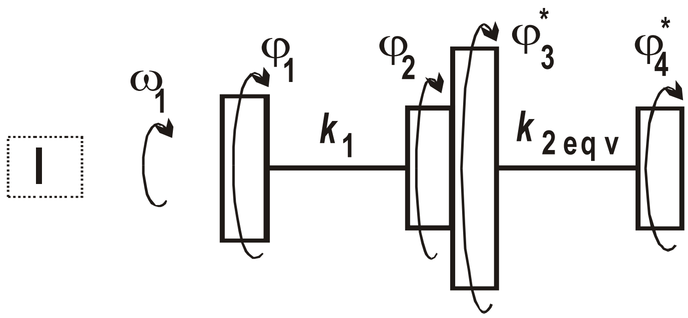

Fig. 2 The model to calculate the equivalent rigidity of the driven shaft

The deformation potential energy of the shaft **II** for real system (fig. 1) can be written as:

$$V = \frac{1}{2} k_2 (\varphi_3 - \varphi_4)^2 \tag{2}$$

For the same shaft **II**, the potential energy, on the basis of the equivalent model from fig. 2, has the expression:

$$V_{eqv} = \frac{1}{2} k_{2eqv} \left(\varphi_3^* - \varphi_4^*\right)^2 \tag{3}$$

Equating the expressions (2) and (3) of the potential energy of the shaft **II**, we obtain:

$$\frac{k_2}{k_{2eqv}} = \frac{\left(\varphi_3^* - \varphi_4^*\right)^2}{(\varphi_3 - \varphi_4)^2} \tag{4}$$

Taking into consideration that, **<u>in steady-state conditions</u>**, the working device moment (of resistance) is equal to the elastical moment from the driven shaft, it may be written as follows [12] [13] [14]:

●for real system

$$M_{WD} = k_2 (\varphi_3 - \varphi_4) \tag{5}$$

●for equivalent system

$$M_{WDeqv} = k_{2eqv} \left(\varphi_3^* - \varphi_4^*\right) \tag{6}$$

Dividing the relations (5) and (6), it is obtained:

$$\frac{\varphi_3^* - \varphi_4^*}{\varphi_3 - \varphi_4} = \frac{k_2}{k_{2eqv}} \frac{M_{WDeqv}}{M_{WD}} \tag{7}$$

Considering the relation (4), we can write

$$\sqrt{\frac{k_2}{k_{2eqv}}} = \frac{k_2}{k_{2eqv}} \frac{M_{WDeqv}}{M_{WD}} \tag{8}$$

or

$$\frac{k_{2eqv}}{k_2} = \left(\frac{M_{WDeqv}}{M_{WD}}\right)^2 \tag{9}$$

From (9), we can write:

$$k_{2eqv} = k_2 \left(\frac{M_{WDeqv}}{M_{WD}}\right)^2 \tag{10}$$

In order to estimate the fraction between working device moments from (10), it has to be written the working device power both for real system and for equivalent system [15] [16] [17]. If there are no mechanical losses in the gearing **2-3**, all power from the motor goes to the working device, that’s why it may be written

$$P = M_{WDeqv} \cdot \omega_1 = M_{WD} \cdot \omega_4 \tag{11}$$

From the relation (11), we can write the fraction between the working device equivalent moment and the working device real moment as follows:

$$\frac{M_{WDeqv}}{M_{WD}} = \frac{\omega_4}{\omega_1} \tag{12}$$

Since, **in steady-state conditions**, the average angular speed of the working device ω_4 is equal to angular speed of the wheel gear **3** (ω_3) and the average angular speed of the motor ω_1 is equal to the angular speed of the wheel gear **2** (ω_2), the relation (12) becomes:

$$\frac{M_{WDeqv}}{M_{WD}} = \frac{\omega_4}{\omega_1} = \frac{\omega_3}{\omega_2} = \frac{1}{i} \tag{13}$$

Taking into consideration relation (13), the calculus formula for the rigidity coefficient of the shaft **II** on the motor shaft is:

$$k_{2eqv} = \frac{k_2}{i^2} \tag{14}$$

2.2.Calculus of the equivalent rigidity on the driven shaft

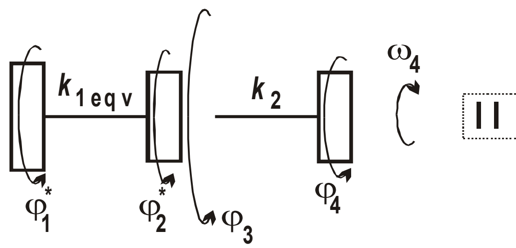

Fig. 3 The model to calculate the equivalent rigidity of the driving shaft

Figure 3 shows the calculus model of the rigidity equation on the axle of the driven shaft **II**. The significance of the notations is as follows:

► k_{1eqv} - the equivalent rigidity coefficient of the driving shaft

► φ_1^*, φ_2^* - the equivalent angular rotations of the driving shaft's terminations

► ω_4 - the average angular speed of the driving shaft (**in steady-state conditions**)

As in §2.1, the deformation potential energy of the driving shaft **I** can be written like this

$$V = \frac{1}{2} k_1 (\varphi_1 - \varphi_2)^2 \tag{15}$$

For the same shaft **I**, the potential energy calculated with the equivalent rigidity k_{1eqv} and angular deflections φ_1^*, φ_2^* is as follows:

$$V_{eqv} = \frac{1}{2} k_{1eqv} \left(\varphi_1^* - \varphi_2^*\right)^2 \tag{16}$$

Since the potential energy of the shaft **I** has to remain the same after the process of equation, from relations (16) and (15) it can be written the fraction between the rigidities as follows:

$$\frac{k_1}{k_{1eqv}} = \frac{\left(\varphi_1^* - \varphi_2^*\right)^2}{(\varphi_1 - \varphi_2)^2} \tag{17}$$

Assuming that, **in steady-state conditions**, the motor has the same average angular speed like the wheel gear **2** and the working device has the same average angular speed like the wheel gear **3**, that meaning $\omega_1 = \omega_2$ and $\omega_3 = \omega_4$, the motor moment has to be equal to the elastical torsion moment from the shaft **I**. In consequence, it can be written:

►for the real system

$$M_M = k_1 (\varphi_1 - \varphi_2) \tag{18}$$

►for the system with equivalent rigidity

$$M_{Meqv} = k_{1eqv}\left(\varphi_1^* - \varphi_2^*\right) \tag{19}$$

Dividing the relations (24) and (25) it is obtained

$$\frac{\varphi_1^* - \varphi_2^*}{\varphi_1 - \varphi_2} = \frac{k_1}{k_{1eqv}} \frac{M_{Meqv}}{M_M} \tag{20}$$

Considering the fraction of the rigidities done by (17), the relation (20) becomes

$$\sqrt{\frac{k_1}{k_{1eqv}}} = \frac{k_1}{k_{1eqv}} \frac{M_{Meqv}}{M_M}, \tag{21}$$

or

$$\frac{k_{1eqv}}{k_1} = \left(\frac{M_{Meqv}}{M_M}\right)^2 \tag{22}$$

From the relation (22),we may say that the equivalent rigidity of the shaft **I** is function of the fraction of the motor moments as follows:

$$k_{1eqv} = k_1\left(\frac{M_{Meqv}}{M_M}\right)^2 \tag{23}$$

The fraction between motor moments from (26) can be determined by writing the motor power both for real system and for equivalent system.

Considering the gear step **2-3** as ideal, all power from the motor goes to the working device, that's why we may write:

$$P = M_M \cdot \omega_1 = M_{Meqv} \cdot \omega_4 \tag{24}$$

From (24), we can write:

$$\frac{M_{Meqv}}{M_M} = \frac{\omega_1}{\omega_4} \tag{25}$$

Since $\omega_1 = \omega_2$ and $\omega_3 = \omega_4$, the fraction between the motor moments can be written function of the gear ratio as follows:

$$\frac{M_{Meqv}}{M_M} = \frac{\omega_1}{\omega_4} = \frac{\omega_2}{\omega_3} = i \tag{26}$$

In this case, the calculus formula of the equivalent rigidity of the driving shaft on the driven shaft axle is:

$$k_{1eqv} = k_1 \cdot i^2 \tag{27}$$

3.CALCULUS OF THE EQUIVALENT RIGIDITIES OF THE MECHANISM'S SHAFTS WITH GEARINGS

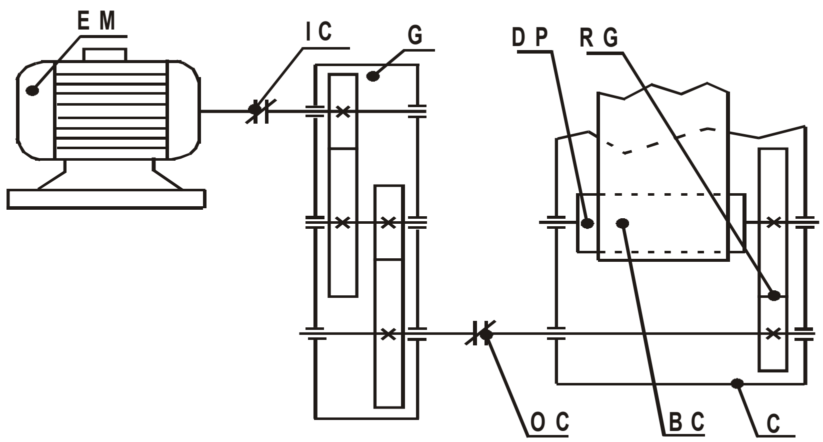

Fig. 4 The principle model of a belt conveyor
EM-electromotor, **IC**-inside coupling, **OC**-outside coupling, **G**-gear reducer unit
C-case, **RG-**reducing gear, **BC**-belt conveyor, **DP**-drive pulley

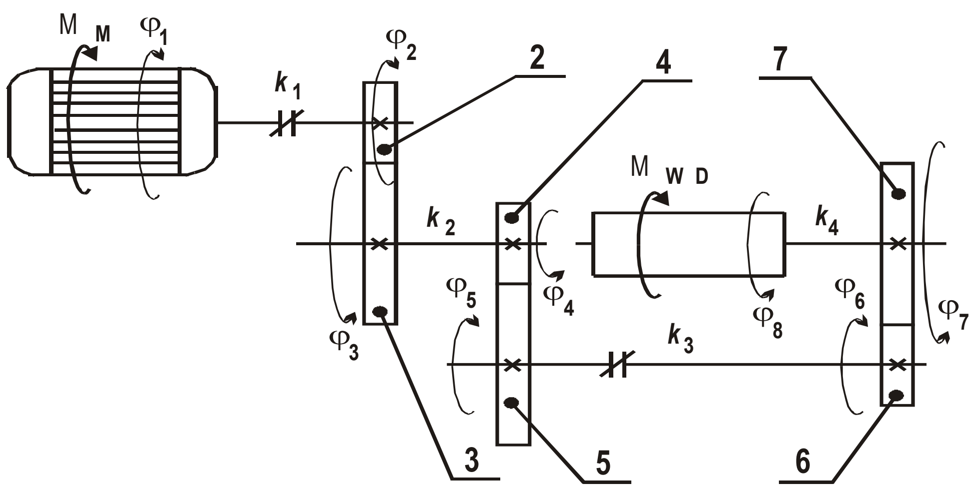

Fig. 5 The skeleton diagram for the belt conveyor

To exemplify the method of the rigidity equation for the mechanism with multiple gearing, it is considered the driving system for a belt conveyor from fig. 4. The skeleton diagram of the acting device is shown in fig. 5, where **2**, **3**, **4**, **5**, **6** and **7** are the gearing wheels of the transmission. We consider as known the ratio of the gearings as follows:

-gearing **2-3** → i_1

-gearing **4-5** → i_2

-gearing **6-7** → i_3

Using the calculus relationships determinated in §2, we will equate the shafts' rigidities both on the electromotor axle and on the drive pulley axle.

Figure 6 shows the calculus diagram of the equivalent rigidities on electromotor axle, where the formulae for the equivalent moments of inertia can be taken from [2] [3] [8]. The calculus relationships used in this case are (14). The equivalent rigidities on the electromotor axle for the shafts 2, 3 and 4 are shown in the table 1 (column 1).

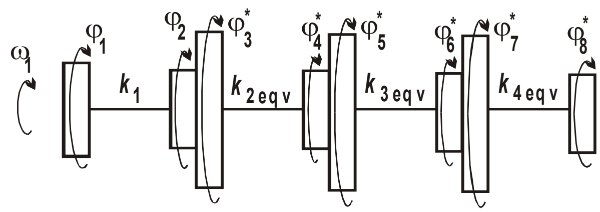

Fig. 6 The calculus diagram for the equivalent rigidities on the electromotor axle shaft

Table 1

Equivalent rigidities - Ideal gearings ($\eta = 1$)

on the driving axle shaft	on the driven pulley axle shaft
$k_{2eqv} = \dfrac{k_2}{i_1^2}$	$k_{1eqv} = k_1 i_3^2 i_2^2 i_1^2$
$k_{3eqv} = \dfrac{k_3}{i_1^2 i_2^2}$	$k_{2eqv} = k_2 i_3^2 i_2^2$
$k_{4eqv} = \dfrac{k_4}{i_1^2 i_2^2 i_3^2}$	$k_{3eqv} = k_3 i_3^2$

Figure 7 shows the calculus diagram of the equivalent rigidities on the drive axle shaft, where, like for §3.1, the equivalent moments of inertia can be taken from [2] [3] [8]. The calculus relationships used in this case are (27) for ideal gearings. Table 1 (column 2) shows the equivalent rigidities on the drive pulley axle for the shafts 1, 2 and 3.

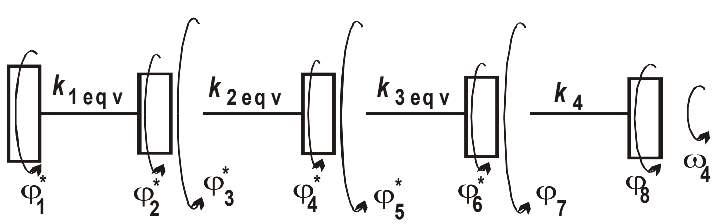

Fig. 7 The calculus diagram for the equivalent rigidities on the driven pulley axle shaft

4. CONCLUSIONS

The methods and calculus formulae presented in this study are useful both to design engineers and to dynamics experts, as well as to students, candidates for master's and doctor's degree.

Regarding the ratio i of the gearings, we may draw some conclusions about its influence on the equivalent rigidities:
1^{0}if $i = 1$ (for changing the sense of rotation only), the equivalent rigidities stay unchanged;
2^{0}if $i > 1$ (reduced step gearing), the rigidity of the driving shaft (on the driven shaft axle) is amplified by i^2 and the rigidity of the driven shaft (on the driving shaft axle) is divided by i^2;
3^{0}if $i < 1$ (amplifier step gearing) the rigidity of the driving shaft (on the driven shaft axle) is divided by i^2 and the rigidity of the driven shaft (on the driving shaft axle) is amplified by i^2.

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